PC– 360 CV-19

M.A./M.Sc. Mathematics (III SEM.)
Examination Dec-2020
Compulsory
Paper-II
INTEGRATION THEORY AND FUNCTINAL ANALYSIS-I

Time: Three Hours] [Maximum Marks: 80
[Minimum Pass Marks: 29

Note: Answer from Both the Section as Directed. The figures in the right hand margin indicate marks.

Section –A

1. Answer the following questions: 1X10
   a. A signed measure μ is totally finite if E is measurable and__________
   b. The measure of every______________is Zero.
   c. Every________________ of a negative set is negative
   d. The Hahn decomposition is not_______________
   e. The radon-Nikodym derivative $dv/d\mu$ is unique___________with respect to μ
   f. A necessary and sufficient condition that $A_1 x A_2$ be a σ –algebra is that______________
   g. If f is a function of bounded variation, then____________exists almost everywhere.
   h. Every absolutely continuous function is differentiable
   i. Every absolutely continuous function f(x) is an___________ of its own derivatives.
   j. If the derivative of two absolutely continuous functions then the function_____________
2. Answer the following question 2X5
   a. Define signed measure
   b. State Hahn decomposition theorem
   c. Define product measure
   d. Define total variation of a function
   e. Define Baire measure.

Section –B

Answer all questions 12X5

3. State and prove Radon Nikodym theorem

OR

Show that the Radon Nikodym theorem for a finite measure μ implies the theorem for σ-finite measure μ.

4. State and prove Lebesgue Decomposition theorem

OR

Show that if V is a signed measure such that

$v \perp \mu$ and $v << \mu$, Then V=0

5. State and prove Fubini's Theorem

Or

Show that

$$\int_0^1\int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2}\,dy\,dx \neq \int_0^1\int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2}\,dx\,dy$$

6. State and prove Jordan Decomposition Theorem.

OR

Show that every increasing function on [a,b] is of bounded variation and every function of bounded variation on [a,b] is almost everywhere differentiable on [a,b].

7. a. If μ_0 is a Bair measure and if, for every c in φ

$$>(a) \quad \lambda(c) = inf\{ \mu_0(u_0): C\subset u_0 \in \varphi\}$$

Then prove that λ is regular content.

b. Prove that the union of sequence of outer regular sets is outer regular. Also the union of an increasing sequence of inner regular sets is inner regular

OR

Show that the Borel measure μ is not regular.